INDIAN MARITIME UNIVERSITY

(A Central University Government of India)

END SEMESTER EXAMINATIONS-June/July 2019

B.Tech (Marine Engineering)

Semester-I

Mathematics-I (UG11T2102)

Date: 11-07-2019	**Maximum Marks: 100**
Duration: 3 hrs	**Pass Marks: 50**

i. Use of approved type of scientific calculator is permitted

ii. The symbols have their usual meaning

Section-A **(3x10=30)**

(All Questions are Compulsory)

1(a). Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$

(b). Find the radius of the curvature of any point ($at^2, 2at$) of the parabola $y^2 = 4ax$.

(c). Find the first and second partial derivatives of $z = x^3 + y^3 - 3axy$.

(d). Find the asymptote of the curve $x^2y^2 - x^2y - xy^2 + x + y + 1 = 0$.

(e). If $u = \tan^{-1}\frac{x^3 + y^3}{x - y}$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

(f). Prove that $\Gamma(n+1) = n\Gamma(n)$.

(g). Find an unit vector normal to the surface $xy^3z^2 = 4$ at the point (-1,-1,2).

(h). A vector field is given by $\vec{A} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$. Show that the field is irrotational.

(i). Find the eigen value of $\begin{pmatrix} 1 & -1 \\ 2 & 4 \end{pmatrix}$.

(j). Evaluate $\int_c \frac{z^2 - z + 1}{z - 1} dz$ where c is the circle $|z| = 1$.

PART B **(5x14=70 Marks)**

(Answer any five questions)

2(a). If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, prove that $(x^2 - 1)y_{n+2} + (2n+1)xy_{n+1} + (n^2 - m^2)y_n = 0$. (7)

(b). Find the asymptote of the curve $x^3 + 3x^2 y - 4y^3 - x + y + 3 = 0$. (7)

3(a). A window has a form of a rectangular surmounted by a semi-circle. If the perimeter is 40 feet, find its dimension so that the greatest amount of light may be admitted. (7)

3(b). If z is a homogeneous function of degree n in x and y, show that

$$x^2 \frac{\partial^2 z}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} = n(n-1)z \text{.} \quad (7)$$

4(a). Prove that $\beta(m, n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$ (7)

4(b). Find the volume of a sphere of radius a. (7)

5(a). Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. (7)

5(b). Evaluate $\int_{y=0}^{1} \int_{x=y}^{1} e^{x^2} dxdy$ by changing the order of integration. (7)

6(a). Find the work done in moving a particle in the force field

$F = 3x^2 i + (2xz - y) j + zk$,(i,j,k are the unit vectors along x,y and z axis respectively) along the straight line from (0,0,0) to (2,1,3). (7)

6(b). Prove that $div(r^n R) = (n+3)r^n$. Hence show that $\frac{R}{r^3}$ is solenoidal. (7)

7(a). Solve the equations $3x + y + 2z = 3, 2x - 3y - z = -3, x + 2y + z = 4$ by

By (i) Determinant and (ii) Matrix. (7)

7(b). Verify Caley-Hamilton theorem for the matrix

$$\begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$ (7)

8(a). Determine the poles of the function $f(z) = \frac{z^2}{(z-1)^2(z+2)}$ and the

residue at each pole. Hence evaluate $\int_c f(z)dz$, where c is the circle

$|z| = 2.5$. (7)

8(b). Using simplex method solve the following LPP

Max Z= $2x_1 + 3x_2$

Subject to
$$x_1 + x_2 \le 1$$
$$3x_1 + x_2 \le 4$$
$$x_1, x_2 \ge 0$$ (7)

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